CURRENT STUDY-SUBJECTS ON ICE SHELL CONSTRUCTION

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ABSTRACT

This paper describes two fundamental studies for the improvement of the ice shell construction.

The one concerns the minimum thickness of ice dome that is subjected to a concentrated load such as a human live load in the construction process. Regarding the problem as a short-term loading and the elastic behavior of ice, the elastic solution is based on the theory of a spherical shallow shell under a uniformly small circular load. The cases of both single and twin loads on a dome are investigated, assuming that the ice dome will break when the maximum tensile stress reaches a certain value. Estimating that the weight of a human is 100 kg and the allowable stress of the ice is 3 kg/cm^2, it is concluded that the minimum thickness for the ice is 6 cm for spans up to 15 m, and 7 cm for spans between 15 m and 30 m. It is also true for twin loads when the loading distance is 1 m apart.

The other one concerns optimum amount of water to spray in the construction. As the first step toward this end, it describes the basic subject of the freezing phenomenon in which a snow+water layer changes to an ice layer on a horizontal plane. A thermal equation is made: one that takes into considerations the heat loss from convection, evaporation and radiation on the surface, and a simple freezing experiment is executed for the quantitative evaluation of that equation. Based on the results, the average thermal heat transfer coefficient by convection of snow+water is evaluated as $5.55 \cdot 10^{-4}$ cal/(cm^{2}sec^{o}C). The results indicate that the current method of spraying water can be improved; thus shortening the construction period for an ice shell.

1. INTRODUCTION

Ice shells are being used as winter structures in inland Hokkaido, where the volume of snow and sustained sub-freezing temperatures make conditions favorable [1]. Since 1997 in Tomamu, many ice shells have been used each winter for about 3 months as leisure-recreational facilities [2]. The construction method of blowing snow and spraying water onto an air-inflated membrane as formwork has basically constructional rationality [3], having taken only one week to complete ice domes spanning 20 to 30 meters in previous field experiments [4, 5]. The ice shells also have high structural efficiency, because the form is determined automatically under uniform pressure and the membrane stresses are mainly compressive. From experimental studies on the structural engineering problems and the actual applications in ice shell construction conducted in the winter environment of Hokkaido since 1980s, it is being recognized that the ice shell is a practical structure for winter activities in cold and snowy regions. However, there are still engineering problems to be solved and improved concerning design, construction, structural safety and maintenance in order to realize a more reliable structure and advance the application further.

This paper describes two current study-subjects with regards to engineering problems in the ice shell construction as seen in Fig.1. The one concerns minimum thickness of ice dome. It describes a numerical investigation of structural safety where the ice dome is subjected to a concentrated load such as a human live load on the apex. Regarding the problem as a short-term loading and the elastic behavior of ice, the elastic solution is based on the theory of a spherical shallow shell under a uniformly small circular load. The cases of both single and twin loads on a dome are investigated, assuming that the ice dome will break when the maximum tensile stress reaches a certain value. The other one concerns a fundamental study on amount of water to spray in the ice shell construction. Knowing the optimum amount of water to spray is an important factor in the rationality of ice shell construction. As the first step toward this end, it concerns the basic subject of the freezing phenomenon in which a snow+water layer changes to an ice layer on a horizontal plane. A thermal equation is made: one that takes into considerations the heat loss from convection, evaporation and radiation on the surface, and a simple freezing experiment is executed for the quantitative evaluation of that equation.

Fig.1 Spraying water on ice dome under construction

2. MINIMUM THICKNESS OF ICE DOME

Anyone occasionally stands up on the ice dome under various situations like the watering work in the construction process and the snow removal in the maintenance management process. In such situation, the minimum thickness of the ice should be determined to ensure safety. A local bending stress occurs at the vicinity of the concentrated loading point and may cause a fatal fracture in the case of a brittle material such as ice. Therefore the structural safety of the ice dome must be investigated.

This section describes a numerical investigation of structural safety when the ice dome is subjected to a concentrated load such as a human live load on the apex. Regarding the problem as a short-term loading and the elastic behavior of ice, the elastic solution is based on the theory of a spherical shallow shell under a uniformly small circular load. The cases of both single and twin loads on a dome are numerically investigated, assuming that the weight of a human is 100 kg and the ice dome will break when the tensile stress reaches 3 kg/cm^2.

2-1. Elastic Solution of Uniformly Circular Load

The circumferential stress on the inner surface, σ_t, is given by Eq.(1) where a shallow spherical shell is subjected to a uniformly circular load as shown in Fig.2 [6].

$$\sigma_t = \frac{M_t}{\left(\frac{h^2}{6}\right)} + \frac{N_t}{h} = f_s(x)\left(\frac{P_t}{h^2}\right),$$

$$f_s(x) = \begin{cases} = -\left(\frac{6}{\pi}\right)\left[(\frac{\ker'\alpha}{\alpha})\{-\nu bei x + (1-\nu)\frac{ber' x}{x}\} - (\frac{kei'\alpha}{\alpha})\{\nu ber x + (1-\nu)\frac{bei' x}{x}\}\right] \\ -\left(\frac{\sqrt{12(1-\nu^2)}}{\pi}\right)\left\{(\frac{kei'\alpha}{\alpha})(-bei x - \frac{ber' x}{x}) + (\frac{\ker'\alpha}{\alpha})(ber x - \frac{bei' x}{x}) + \frac{1}{2\alpha^2}\right\}, & 0 \le x \le \alpha \\ = -\left(\frac{6}{\pi}\right)\left[(\frac{ber'\alpha}{\alpha})\{-\nu kei x + (1-\nu)\frac{\ker' x}{x}\} - (\frac{bei'\alpha}{\alpha})\{\nu \ker x + (1-\nu)\frac{kei' x}{x}\}\right] \\ -\left(\frac{\sqrt{12(1-\nu^2)}}{\pi}\right)\left\{(\frac{bei'\alpha}{\alpha})(-kei x - \frac{\ker' x}{x}) + (\frac{ber'\alpha}{\alpha})(\ker x - \frac{kei' x}{x}) - \frac{1}{2x^2}\right\}, & x \ge \alpha \end{cases} \quad (1)$$

Where $P_t = \pi a^2 q$, P_t is the total load, $x = \frac{r}{l}$, $l^2 = \frac{Rh}{\sqrt{12(1-\nu^2)}}$, l is characteristic length, $\alpha = \frac{a}{l}$, $' = \frac{d}{dx}$, *ber;bei,ker* and *kei* are kelvin functions. R is the radius in the spherical shell, $D\left(=\frac{Eh^3}{12(1-\nu^2)}\right)$ is flexural rigidity of the plate, h is the shell thickness, E is Young's modulus, ν is Poisson's ratio and q is the constant load per unit area over the circle with radius a.

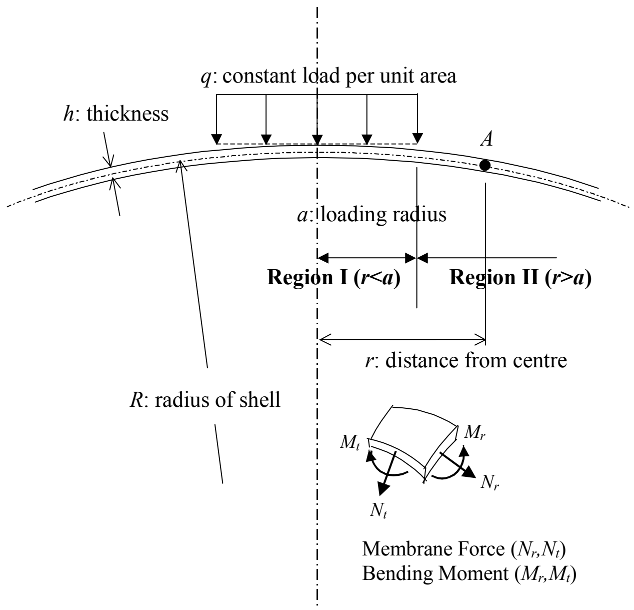

Fig. 2 Spherical shallow shell under uniformly circular load

2-2. Single Load

A single uniformly circular load is discussed here. The maximum tension stress σ_{smax} occurs on the inner surface at the centre of the load and σ_{smax} is given by Eq. (2) using Eq.(1).

$$\sigma_{s\max} = f_s(0)\left(\frac{P_t}{h^2}\right) = \frac{1}{k_s}\left(\frac{P_t}{h^2}\right),$$

where k_s is loading coefficient,

$$k_s = \frac{1}{\left(\frac{3}{\pi}\right)(1+\nu)\left(\frac{kei'\alpha}{\alpha}\right) - \left(\frac{\sqrt{3(1-\nu^2)}}{\pi}\right)\left(\frac{\ker'\alpha}{\alpha} + \frac{1}{\alpha^2}\right)} \quad (2)$$

Fig.3 shows the relationship between k_s and α where α<0.8. The k_s can be very closely approximated by the linear function of α between 0.15 and 0.35. Eq.(3) shows the straight line connecting (α=0.2, k_s=0.8467) and (α=0.3, k_s=1.0683) where ν=0.3.

$$k_{sA} = 0.4035(1 + 5.492\alpha) \quad (3)$$

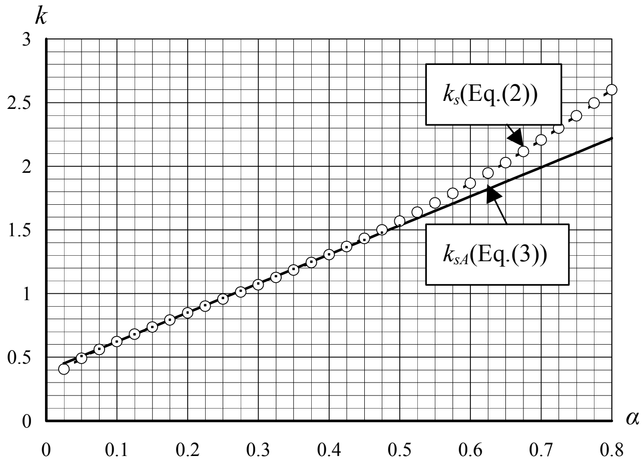

Fig.3 Comparison between k_s and k_{sA}

2-2-1. Flexural Strength of Ice

To test the flexural strength of an ice beam, a short-term bending experiment was conducted. As in the construction of ice shells, the ice beams were made by freezing a mixture of snow and water outdoors in natural sub-freezing temperatures. The approximate dimensions were 50 cm in length (L), 8 cm in width (b) and 4 cm in thickness (h).

The flexural strength σ_f of the simply supported, centre-loaded ice beams was computed according to the relationship: $\sigma_f = \frac{3P_f L}{2bh^2}$ Where P_f is the force at failure. P_f was electrically measured with a 100 kg capacity load cell. Fig.4 shows the relationship between σ_f and ρ_i, the density of the ice. Based on the result, 10 kg/cm^2 was the smallest value in this experiment.

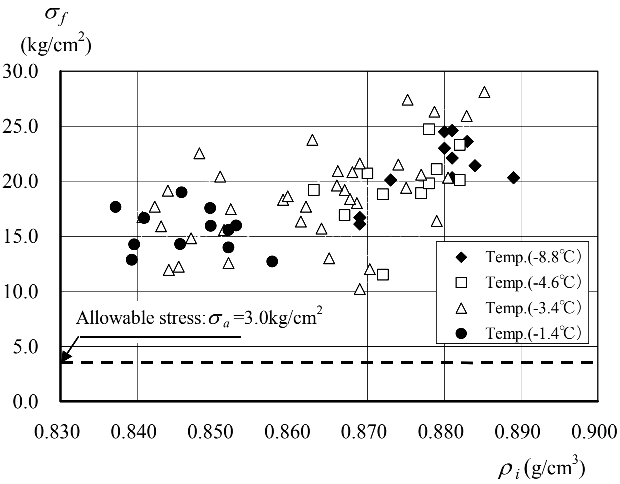

Fig.4 Flexural strength of ice beam

2-2-2. Discussions

Using Eq.(3), structural safety when human weight of 100 kg is loaded on the ice dome is numerically examined as follows:

Assuming that the shoe size is 10 cm×30 cm, the equivalent radius a is estimated at 10 cm for the same area.

Using ν=0.3, $\alpha = \frac{a}{l} = \frac{a}{\sqrt{Rh}}\sqrt[4]{12(1-\nu^2)} = \frac{18.18}{\sqrt{Rh}}$ Furthermore, assuming that the open angle of the dome is 120°, the final expression of α becomes $\alpha = \frac{23.93}{\sqrt{D_s h}}$, where D_s is the diameter of the dome at the base. Table 1 shows the numerical results where D_s=15 m and 30 m. Here, the compressive membrane stress σ_c(kg/cm^2) is computed 0.37 kg/cm^2 for D_s=15 m and 0.74 kg/cm^2 for D_s=30 m based on the membrane theory in shell under the gravity load and these σ_c are neglected in the Table 1 for the safety side of the evaluation. According to the experiment of ice beams mentioned above, the short-term flexural strength of the ice is 10 kg/cm^2 at the lowest. Because good quality of ice is produced artificially by the careful application of snow by blowing and water by spraying onto a pneumatic membrane, the density of the ice becomes about 0.85 g/cm^3 polycrystalline ice. If the allowable stress is evaluated at 3.0 kg/cm^2 and σ_{smax} does not exceed 3 kg/cm^2, it is judged that the dome is thick enough to have structural safety against a short-term flexural failure. As a result, the minimum thickness of the ice is 6 cm for spans up to 15 m, and 7 cm for spans between 15 m and 30 m at the base.

Table 1 Numerical result (P_t=100kg)

D_s(m)	h(cm)	α	k_{sA}	σ_{smax}(kg/cm^2)
15	5	0.276	1.016	3.94
	6	0.252	0.962	2.89
30	6	0.178	0.799	3.48
	7	0.165	0.769	2.65

2-3. Twin Loads

The problem where the ice dome is subjected to the same size and weight of two circular uniform loads keeping the distance, s, between the central points, is discussed here. According to the elastic solution, the tensile stress on the inner surface of the shell under single loading has the following relationship $\sigma_t \geq \sigma_r$, where σ_t and σ_r are circumferential and radial stress, respectively, and the both are equal directly below the centre point of the load. The σ_s or $f_s(x)$, is already given by Eq.(1).

Fig.5 shows two curves of f_s: one $f_s(x)$ and the other $f_s(x-s)$. The aim of the subject here is to find out the exact maximum value of $f_d(x)(=f_s(x)+f_s(x-s))$ where $0 \leq x \leq s/2$. However, comparing the values of both ends,

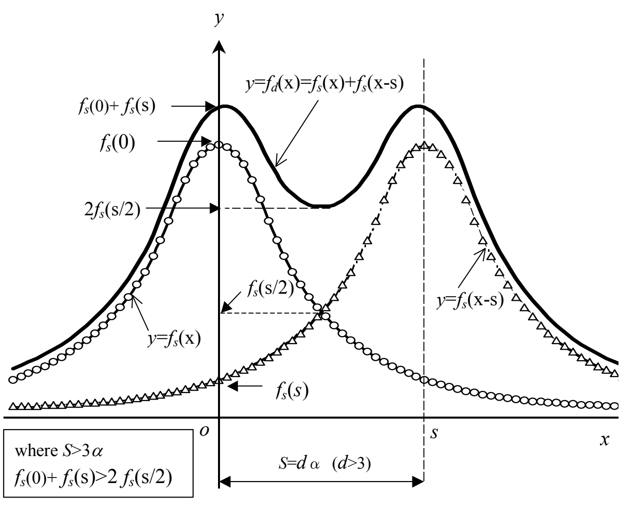

Fig.5 Stress distribution under twin loads

$f_d(0)(=f_s(0)+f_s(s))$ *and* $f_d(\frac{s}{2})(=2f_s(\frac{s}{2}))$, the larger one is considered the maximum value f_{dmax} in the twin loading problem. As s is clearly larger than 2α, therefore, the following Eqs.(4.a-c) are given according to the range of s.

$$\text{where} \quad 2\alpha < s < s_{eq}, \quad f_d(\frac{s}{2}) > f_d(0) \qquad (4.a) \qquad \text{where} \quad s = s_{eq}, \quad f_d(\frac{s}{2}) = f_d(0) \qquad (4.b)$$

$$\text{where} \quad s > s_{eq}, \quad f_d(\frac{s}{2}) < f_d(0) \qquad (4.c)$$

In solving Eq.(4.b), the s_{eq}–α curve is found in the region between $s=2\alpha$ and $s=3\alpha$. Therefore, where $s = d\alpha \ (d \geq 3)$, $f_d(\frac{s}{2}) < f_d(0)$ is found. $f_d(0)$ is given through Eq.(5).

$$f_d(0) = -\left(\frac{6}{\pi}\right)[(\frac{ber'\alpha}{\alpha})\{-\nu kei(d\alpha)+(1-\nu)\frac{\ker'(d\alpha)}{(d\alpha)}\} - (\frac{bei'\alpha}{\alpha})\{\nu\ker(d\alpha)+(1-\nu)\frac{kei'(d\alpha)}{(d\alpha)}\}]$$
$$-\left(\frac{\sqrt{12(1-\nu^2)}}{\pi}\right)\left\{(\frac{bei'\alpha}{\alpha})\left(-kei(d\alpha)-\frac{\ker'(d\alpha)}{(d\alpha)}\right)+(\frac{ber'\alpha}{\alpha})\left(\ker(d\alpha)-\frac{kei'(d\alpha)}{(d\alpha)}\right)-\frac{1}{2(d\alpha)^2}\right\} \qquad (5)$$
$$+\left(\frac{3}{\pi}\right)(1+\nu)\left(\frac{kei'\alpha}{\alpha}\right)-\left(\frac{\sqrt{3(1-\nu^2)}}{\pi}\right)\left(\frac{\ker'\alpha}{\alpha}+\frac{1}{\alpha^2}\right)$$

In accordance with the previous single loading problem, the loading coefficient $k_d = \frac{1}{f_d(0)}$ is approximated through a linear equation of α as shown in Eq.(6).

$$k_{dA} = A(d)\alpha + B(d) \qquad (6)$$

$A(d)$ and $B(d)$ of Eq.(6) are shown in Fig.6a and Fig.6b, respectively, and $\lim_{d\to\infty} A(d)$ *and* $\lim_{d\to\infty} B(d)$ are 2.2160 and 0.4035, respectively which agree with the coefficients of Eq.(3). Based on the approximate curve in Fig.(6a,b) and Eq.(6), a numerical result is presented in Table 2. As seen in the table, it is concluded that the minimum thickness for the ice is 6 cm for spans up to 15 m, and 7 cm for spans between 15 m and 30 m. It is also true for twin loads when the loading distance is 1 m apart.

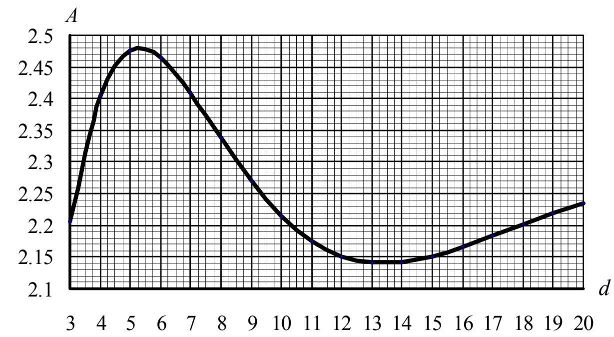

Fig.6a ***A-d*** **curve**

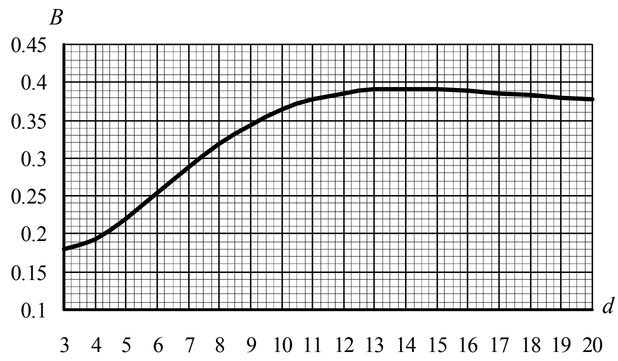

Fig. 6b ***B-d*** **curve**

Table 2 Numerical result of twin loads (P_t=100kg)

D_s(m)	h(cm)	α	d	$A(d)$	$B(d)$	k_{dA}	σ_{dmax}(kg/cm^2)
15	6	0.252	5	2.4766	0.22036	0.844	3.29
			10	2.2153	0.36382	0.922	3.01
30	7	0.165	5	2.4766	0.22036	0.629	3.24
			10	2.2153	0.36382	0.729	2.80

3. OPTIMUM AMOUNT OF WATER TO SPRAY IN CONSTRUCTION

3-1. Current Method

Blowing snow and spraying water onto the ice shell produce a new thin layer of ice, as shown in Fig.7. The snow is blown at intervals of 45 to 90 minutes, with each application taking from 10 to 30 minutes, depending on the size of the shell and the meteorological conditions. The snow crushed with a snow blower is a type of sintering snow, with a density and thickness per application of 0.4~0.5 g/cm^3 and 1 cm or less. As the snow is being blown, water is continuously sprayed through an adjustable nozzle. The snow absorbs the sprayed water and keeps some of it from running down the sides. Since snow is a form of ice, it is natural that snow+water freezes faster than water alone; therefore, it is possible to complete a dome in less time and at higher temperatures by using snow in the construction. Experience shows that the thickness of the ice increases by about 1 cm every 90 minutes when the outside temperature is approximately –10^{o}C, and its density is in the range of 0.83 to 0.88 g/cm^3. The thin snow layer must be saturated, but the amount of water needed varies with the air temperature. The amount of sprayed water in actual construction has, for a 10-m dome, the broad range, 14 to 40 *l*/minute. This information is based on the findings from construction experience up to the present. Considering that the freezing phenomenon from snow+water layer to ice layer can be captured from an engineering standpoint as a thermal problem, it is an important subject to examine in order to determine the optimum amount of water to spray for the constructional rationality.

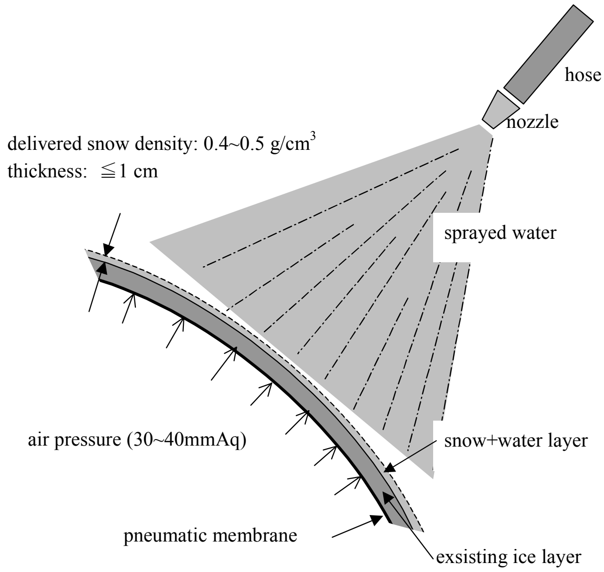

Fig. 7 Application of snow and water

3-2. Mathematical Formulation

As shown in Fig.7, there is a snow layer with a thickness of 1 cm and a density of ρ_s (g/cm^3), on the previously produced ice layer. The snow layer becomes a snow+water layer when water is sprayed onto it. The amount of water sprayed must be adjusted corresponding to the variation in water retentivity caused by the inclination of the dome's curved surface; therefore, it must be determined according to the angle of gradient θ shown in Fig.8. This study, as a start, deals with the freezing process on a horizontal plane θ=0^o. In order to resolve the problem, the following assumptions are made: right after the spraying of water, the temperature of snow+water layer is 0^{o}C; there is no change in the weight of either the snow or the water, and there is no heat transfer between the snow and the water during the freezing process. Then, the freezing phenomenon of the snow+water layer can be simplified to the freezing phenomenon of just the water in the layer. Since, according to experience, the temperature of water is in the range of 0 to 5^{o}C and that of the snow is close to the air temperature which is below –10^{o}C, it should be acceptable to make the above assumption when water spray is regulated for greatest efficiency. When the produced ice density is ρ_i(g/cm^3), the amount of water in cm h_w per 1 cm snow thickness is approximately given by Eq.(7).

$$h_w = (\rho_i - \rho_s) / 0.917 \qquad (7)$$

Where the density of pure ice is 0.917 g/cm^3 and the density of the snow blown is ρ_s. Therefore, the problem to be solved is stated as follows: **Find the freezing time for the 0^{o}C water layer with thickness of h_w(<1 cm) to turn to ice.**

The thickness of the snow+water layer is less than 1 cm and the freezing time is within 90 minutes; therefore, the problem can be treated as a stationary thermal process. As seen in Fig.8, the freezing phenomenon in the snow+water layer generally begins at both the inner and outer surfaces. However, the inner-surface heat loss from conduction is negligible compared to that of the outer surface. The total heat loss from the outer surface is stated as follows:

$$F = C+E+R \qquad (8)$$

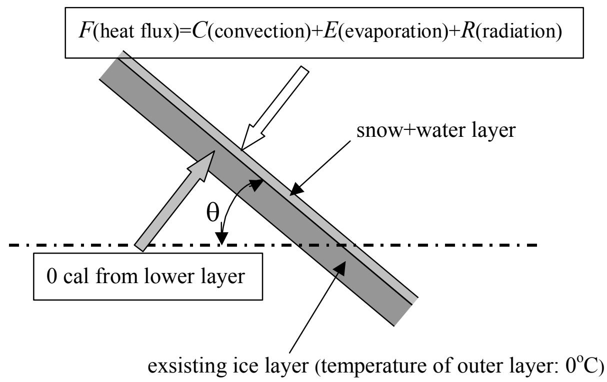

Fig.8 General model for thermal exchange

Where C, E and R are the sensible heat flux, evaporation and long-wave radiation, respectively. The sensible heat flux, C cal/(cm^{2}sec) is given by Eq.(9).

$$C=\alpha(T_w-T_a)=-\alpha T_a \quad (9)$$

Where T_a is the outside air temperature (°C); T_w is the surface temperature of the snow+water layer, which is 0°C; and α (cal/(cm^{2}sec°C)) is the coefficient of heat transfer by convection, given by a linear equation of wind velocity. Next, the evaporative heat flux E (cal/(cm^{2}sec)) is given by the following:

$$E=\beta\times(4.58-e) \quad (10)$$

where β and e are, respectively, the coefficient of heat transfer by evaporation (cal/(cm^{2}secmmHg))and the vapor pressure (mmHg) at T_a. Assuming that C and E have the same divergence mechanism expressed by a bulk equation on a water surface and snow surface, the following relationship between α and β can be obtained.

$$\beta=2.05\times\alpha \quad (11)$$

Then, assuming that the humidity and air temperature, being consistent with the meteorological data of the inland Hokkaido area where the ice shells are constructed, is 80 % and in the range of −10 and −20 °C, e is approximated by Eq. (12).

$$e= 2.683 + 0.0966\,T_a \quad (12)$$

Therefore, $$C+E= (3.89-1.198\,T_a) \quad (13)$$

Since ice shell construction is carried out at night, the heat flux due to radiation under a clear sky, R_o (cal/(cm^{2}sec)) is given by Brunt's Formula.

$$R_o = \delta\sigma\left\{(T_w+273.2)^4-(T_a+273.2)^4\left(p+q\sqrt{e_b}\right)\right\}$$

where δ is ice emissivity (=1), σ is the Stefan-Boltzman Constant (=1.35·10^{-12}(cal/(cm^{2}sK^4)), p=0.53, q=0.065, T_a is air temperature (°C), T_w is surface temperature (=0°C), e_b is the vapor pressure which is 2.0 mb where T_a is in the range of –10 to –20 °C and humidity is approximately 80 %.
Hence $p+q\sqrt{e_b}=0.62$, and the formula is expressed by the following linear equation (14-a) where T_a is in the range of –10 °C to –20 °C.

$$R_o=10^{-4}\times(29.36-0.576\,T_a)\ \text{(cal/(cm}^2\text{sec))} \quad (14\text{-a})$$

The net long-wave-radiation under a cloudy sky is given by the following equation.

$$R=R_o r\ \text{(cal/(cm}^2\text{sec))} \quad (14\text{-b})$$

where r is the decrease coefficient (r=1−k(n/10)), n is the cloudiness on a scale of 0(clear) to 10(completely clouded), and k corresponds to the cloud height (0.85 – low, 0.7 – mid, 0.2 -- high).
Then, the total heat flux F cal/(cm^{2}sec) is given by Eq.(15), using Eq.(8) to (14-a,b).

$$F=\{\alpha_o(3.89-1.198\,T_a)+(29.36-0.576\,T_a)r\}\times10^{-4}\ \text{(cal/(cm}^2\text{sec))} \quad (15)$$

where $\alpha=\alpha_o\times10^{-4}$.

3-3. Freezing Experiment

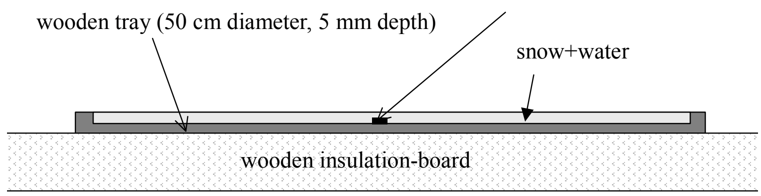

Fig. 9 Method of freezing experiment on horizontal plane

Where the angle of gradient is assumed to be 0 degrees, that is, retentivity of the delivered water is 100 %, the coefficient of heat transfer due to convection α is determined through the following experiments. Wooden trays of 50 cm diameter are prepared as shown in Fig.9. Snow is shaken onto the trays through a sieve (2.5 mm mesh) and leveled with a stainless ruler. After calculating the density of the snow ρ_s (g/cm^3) from the volume of the tray and the weight of the snow, the snow-filled tray is left outdoors for several minutes. Next, the water that is 0°C and of the weight W_w(g) is sprayed evenly on the snow. The temperature of the snow+water layer and the time are recorded automatically in order to find the freezing time, t_f(sec), which is the time span from the application of the water spray to the point at which the temperature begins to make a sudden drop, indicating the completion of freezing process. If the ice density ρ_i is selected, the amount of water W_w (g) is given by Eq.(16).

$$W_w = h_w \times S_s = (\rho_i - \rho_s) \times S_s / 0.917 \qquad (16)$$

Where ρ_s is snow density and S_s is the base area of the tray. For comparison of the freezing behavior, trays filled with water only, and equal in weight to the snow+water, were also prepared.
Assuming that the water W_w(g) of snow+water becomes ice, the heat loss Q_f(cal) is expressed as follows:

$$Q_f = 80W_w \text{(cal)} \qquad (17)$$

Therefore, the coefficient of heat transfer due to convection α can be obtained based on the following equation.

$$Q_f = F \times t_f \times S_s \qquad (18)$$

3-4. Results and Discussions

Fig.10 shows one of the results of the experiment carried out on the rooftop of the three-story administration building of Hokkaido Tokai University between 0 and 4 a.m. on March 13, 2005. Throughout this experiment, the air temperature was in the range of −9 to −10 °C, and the wind velocity was very low (10-minute average wind velocity: 0.18 m/sec, maximum: 0.34 m/sec, minimum: 0.03 m/sec). The coefficient of radiation r =0.15 (k=0.85, n=10). The three kinds of time-temperature curves shown in Fig.10 are the outside air (○, T_a), A(snow+water, T_A,□) and B(water, T_B, Δ). Here, the horizontal axis is time (minutes) and the vertical axis is temperature (°C). In this example, the weight of the snow and of the water for model A snow+water was 390 g and 489g, respectively. 879 g of water, which was the same as the total weight in model A, was used in model B (water only). When T_A reached T_a, water was sprayed onto the snow. Immediately after the spraying, T_A rose rapidly and became 0 °C. Then T_A remained at 0 °C for 46 minutes, until the rapid drop in temperature that indicates the completion of freezing process. In contrast, T_B remained at 0 °C for 120 minutes before the temperature drop. The ratio of the freezing time per unit weight of water between model A and B is 1.45 (=(120/879)/(46/489)). From this, it can be understood that the coefficient of heat transfer due to convection in model A, 4.59×10^{-4} cal/(cm^{2}sec °C) is 50 % greater than that of model B, 3.00×10^{-4} cal/(cm^{2}sec °C). The coefficients α calculated from the data of these experiments which were diverse, clearly show the general tendency of the mean value of α, which is 5.55×10^{-4} cal/(cm^{2}sec °C) for snow+water, and 3.61× 10^{-4} cal/(cm^{2}sec °C) for water only.

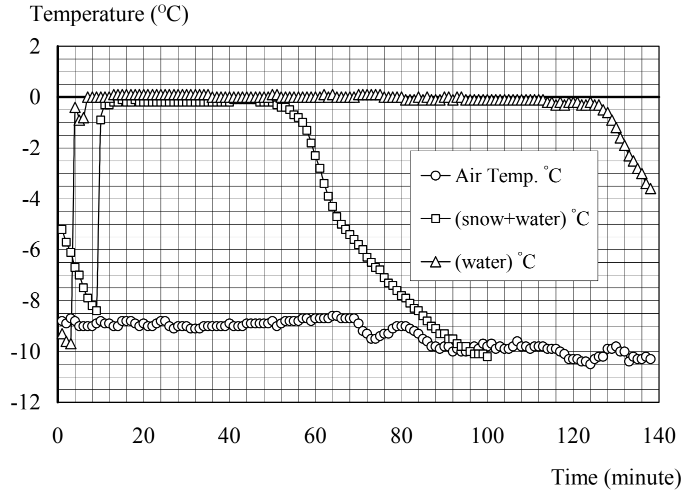

Fig. 10 Example of results (3/13/05,0:00-4:00)

Using the value of α(=5.55×10^{-4}cal/(cm^{2}sec °C)) , a numerical simulation of freezing snow+water on horizontal plane is made where the air temperature is between –10 and –20 °C and sky clear. Substituting α_0=5.55 and r=1 for Eq.(15), the freezing time t_f (sec) for snow of 1 cm thickness is given by Eq.(19).

$$t_f = 8.724 \times 10^5 \frac{(\rho_i - \rho_s)}{(50.95 - 7.225T_a)} \qquad (19)$$

Based on Eq.(19), Fig.11 shows the freezing time t_f (minutes) for snow densities ρ_s (g/cm^3) of 0.4, 0.5 and 0.6 where the ice density ρ_i is 0.85 g/cm^3, and air temperature T_a is in the range of −10 to −20 °C. Eq. (19) indicates that t_f is proportional to ($\rho_i - \rho_s$) and the ratio of freezing time for the same air temperature becomes 9 : 7 : 5 with accompanying times of 41.1, 31.9 and

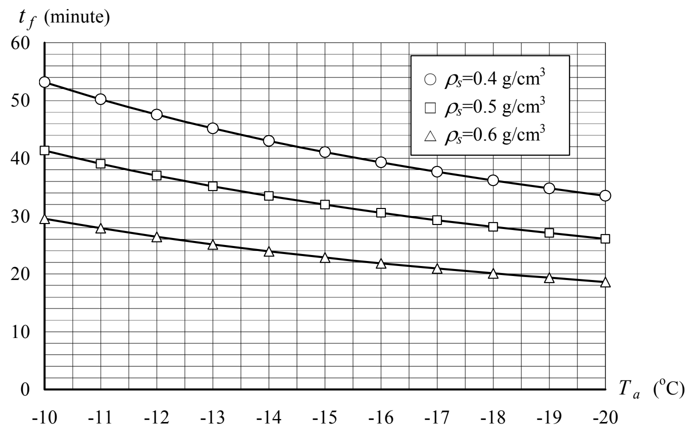

Fig. 11 Freezing time – air temperature

22.8 minutes for snow of 1 cm thickness where T_a is −15 °C. It is clear that the snow density ρ_s greatly affects the freezing time; therefore, knowing a practical method of preparing high- density snow is a very important factor in the rationality of ice shell construction. For example, promoting sintering by crushing the snow in a snow blower during the daytime when the air temperature is relatively high seems to be effective. The standard intensity of delivered water, S_v, is the value of h_w divided by t_f, which is the minimum amount of sprayed water per unit time and area needed for continuous freezing on contact. S_v is given by Eq.(20).

$$S_v = \frac{h_w}{t_f} = \frac{F}{80} = \frac{(50.95 - 7.225T_a)}{80} \times 10^{-4} \qquad (20)$$

According to Eq.(20), S_v has a relationship to T_a but not to ρ_i and ρ_s. Defining total delivered water as the product of the dome's surface area and S_v, the total delivered water refers to the volume of water per unit time necessary to permeate the 1-cm snow layer, and to freeze continuously, over the entire surface of the dome.

Based on the above, the amount of water sprayed in the present method is numerically examined in the case of a 10-m ice dome. The surface area of a 10-m ice dome, for example, is about 100 m^2. Therefore, the total delivered water is calculated as 9.23(liters/minute) when T_a=−10°C to 14.65 liters/minute when T_a=−20 °C. However, not all of the sprayed water becomes ice. Currently, the amount of sprayed water that actually freezes on the dome's surface during construction is 30 % to 70 %, but it is hoped that a method for raising the freezing rate (weight of water that freezes on the shell / weight of delivered water) can be developed in the future. Assuming a rate of 50 %, the two above-mentioned water deliveries are multiplied to become 18.46 liters/minute (−10 °C) ~ 29.30 liters/minute (−20 °C). Considering the temperature of the sprayed water to be in the range of 0 to 5 °C, as stated in 3-2, the freezing time is assumed to be longer than 41 minutes (−10 °C) to 26 minutes(−20 °C)where ρ_s is 0.5 g/cm^3, however, the quantitative evaluation has not yet been verified.

At present, the amount of sprayed water is in the range of 14 to 40 liters/minute for a 10-m dome. However, 40 liters/minute is too much, even where the air temperature is −20 °C. Over-watering seems to be one of the factors in the delay in freezing. Evaluation of the freezing ratio for an inclined surface should be carried out in the future, but the results of this study indicate that, with improvement in the water spraying method, the construction period can be shortened.

4. ENDING REMARKS

According to the translucent thin plate and the peculiar curved surface form, the ice shell creates a fantastically beautiful space in the environment sustained sub-freezing temperatures. The interior space has a brilliant atmosphere with full of natural light in daytime, and the exterior looks like a gigantic chandelier in the dark at night. The ice shell is used as a temporary structure for winter activity inland Hokkaido today. However, as the ice shell is a new type of ice structure since 1980s, the solution and the improvement for various technical problems related to the design, construction, and the control of maintenance are indispensable so that the ice shell may grow up to the common structure in the future. In addition to the studies described in this paper, the subjects such as creep property of the ice in the range of 0 °C to −5 °C, creep deformation of ice shell, construction by the snowmaking machine and ice dome construction on frozen lake are being investigated. On the other side, as the ice shell can be constructed easily at any place if there are the severe cold, snow and water, the shell has a possibility to become a useful structure common in not only inland Hokkaido but also the severe cold regions all over the world such as Canada, Alaska, Northeast China, North Scandinavian, Russia and the South Pole. The author would like to hope for the new development of the usage and the expansion of application in those areas.

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