Proceedings of the 20th International Conference on
Port and Ocean Engineering under Arctic Conditions
June 9-12, 2009
Luleå, Sweden

POAC09-24

SIMPLIFIED FORMULA FOR STATIONARY CREEP DEFLECTION OF ICE DOME

Tsutomu Kokawa

Dept. of Art and Technology, Tokai University, Asahikawa, Japan

ABSTRACT

Based on field experiment data for ice domes spanning 10 to 30-meters*, the creep deflection-time curve is approximated by connecting two lines; the first line represents the stationary stage in which the deflection is a linear function of time and the second represents the accelerating stage in which the deflection rate increases with time up to failure. This paper proposes a simplified formula for computing the creep deflection during the stationary stage based on the data. The derivation of the formula is as follows: assuming ice to be a Newtonian fluid and using the invariant theory for the creep material and the membrane theory for a thin shell, the vertical deflection at the top is derived analytically. The evaluation of the viscosity of the ice is based on the creep deflection data obtained from the field experiments. The average viscosity is computed to be approximately 3500 kgf·cm^{-2}·day. The simplified formula satisfactorily predicts the creep deformation of ice domes when the ice temperature is in the range of 0 ^{o}C to −5 ^{o}C.

*: "30-meters" refers to the diameter of the membrane bag used in the formwork before inflation.

INTRODUCTION

Ice shells, which are thin curved plate-structures made of ice, are being used as seasonal architectural structures in Hokkaido, whose inland snow volume and sustained sub-freezing temperatures in winter make conditions suitable (Kokawa, 2005). The construction method of blowing snow and spraying water onto the pneumatic formwork consisting of a membrane bag and a reticulated rope cover has constructional rationality. A large structure of this kind is able to stand without supporting columns because of its high structural efficiency as a shell. This has been verified in field experiments in which the entire construction period of ice domes spanning 20 to 30 meters was one week or less, and whose structural creep behaviors were stable (Kokawa, 2002a;2002b). The study on ice shells started at the beginning of the 1980s with the goal of constructing ice shells with a 30 m span, which could be used for winter structures (Kokawa and Hirasawa, 1982/1983). Repeated experiments have provided data regarding the constructional and structural performance of the full-scaled models and their practical application as architectural structures. Through these practical studies, the shells have proved themselves to have many advantages such as a short construction period, low construction costs, high structural efficiency, unique shape and environmental compatibility. These features are key to their recognized usefulness as seasonal structures for winter activities in regions with sufficient snow and low temperature. However, for some reasons, the span of these shells has always been limited to 15-meters or less. To date, four field experiments of ice domes spanning 20 to 30-meters have been carried out, but ice domes with a span of 20 meters or more are not in practical use as architectural structures. Investigations have been carried out through the field experiments of ice domes with spans from 10 to 30-meters where the meteorological conditions such as outside air temperature, humidity, radiation, wind, snowfall and the accumulated load of snow on the dome vary. A more reliable structural design method should be based on the creep behavior of ice domes.

According to current rules for the structural design of ice shells; the maximum compressive membrane force in the structure must be less than 1kg/cm^2. Then, compressive strength of the ice produced by blowing snow and spraying water is approximately 40 kg/cm^2 under short-term loading, and the designed shell has the strength to withstand self-weight loading. However, as seen in the past experiments, the ice stays in the narrow temperatures, 0^{o}C to −5^{o}C, and the shell has a tendency to creep easily with time in this range even if the working stress is small. This results in large deformations that end the dome's usability as an architectural structure, and cause instability leading to collapse. Therefore, it is necessary to prepare a rational structural design method that takes into consideration not only stress regulation but also deformation or strain rules in relation to time.

This paper proposes a simplified formula for calculating the creep deflection of an ice dome at the stationary stage. Based on the field experiments of ice domes spanning 10 to 30 meters, the creep deflection-time curve is approximated by connecting two lines; the first line showing the stationary stage where the deflection has a linear function of time, and the second showing the accelerating stage where the deflection rate increases with time up to the failure. Assuming the ice to be a Newtonian fluid, and using the invariant theory of the creep material and the membrane theory of a thin shell, the vertical deflection at the top is derived analytically. The evaluation of the viscosity of the ice is based on the creep deflection data from the field experiments. As a result, the average of viscosity is calculated to be about 3500 kgf·cm^{-2}·day. This leads to a simplified formula that can predict the creep deformation of an ice dome where the ice temperature is in the range of 0^{o}C to −5^{o}C.

RESULTS OF FIELD EXPERIMENTS

The ice shell was realized at the beginning of the 1980s through the rational construction method of blowing snow and spraying water on a formwork consisting of a double-plane membrane bag and reticulated rope cover. The shell can be constructed as long as there is equipment for spraying water, a snow blower, an air blower, a double-plane membrane bag, a reticulated rope cover, snow and water. Also, the shell is environmentally compatible because it simply returns to the earth as water in spring. These factors have facilitated the conducting of field experiments of full-scale ice domes of up to a 30-meter span in inland Hokkaido. The results of past field experiments on ice domes spanning 10, 15, 20 and 30 meters are briefly described here. They were constructed using the above-mentioned method. The structural creep behaviors were investigated first through automatic measurement done by devices bolted to the ice shell at certain points, and then, after their removal, visual observation up to the collapse.

10-m Span Model (Kokawa,1985)

Outline: The first experiment of a 10-m span ice dome using the above-mentioned construction method was carried out at Hokkaido Tokai University Asahikawa campus in 1984. The reticular pattern of the rope cover was a two-way grid whose space was about 1 meter apart. It was completed in the beginning of January. The creep deflection of the dome under a snow loading, which was blown on the dome by a snow blower, was automatically measured during the period of January 20 through March 20. After removal of the measuring devices, the structural behavior up to the collapse was observed visually. This experiment confirmed that the construction method was rational and the structural behavior was efficient.

Geometry of dome: After its completion, three-dimensional coordinates of 21 points on the inner surface were determined. The points were on a nearly spherical surface, whose radius computed by means of least-square fitting, was 765cm. The center height was 222 cm. This dome is shallower than today's dome which usually has a height of approximately 300 cm. The base diameter, computed from the height and the radius, was 1078 cm, which is a strange figure because mathematically it should be smaller than 1000cm. The value of the half-open angle of the spherical dome is 44.8 ^{o} when the base diameter is

1078cm. The thickness was measured at 29 places and the average thickness was 12.5cm. The average ice density was 0.85g/cm^3.

Measurement of temperature: Fig. 1-a shows three temperature-time curves. Tout, Tin and Tice are outside, inside and ice temperature, respectively. Tin and Tice are higher and change more slowly than Tout due to the thermal insulation effect of the accumulated snow on the dome. Tice remained within the range of –5°C to –2°C while Tout remained sub-freezing, which was from the starting point to March 2 as shown in Fig.1-a. Between March 3 and 7, Tout sometimes rose above 0°C; then, after March 11, as Tout often went above, Tin and Tice gradually approached 0°C.

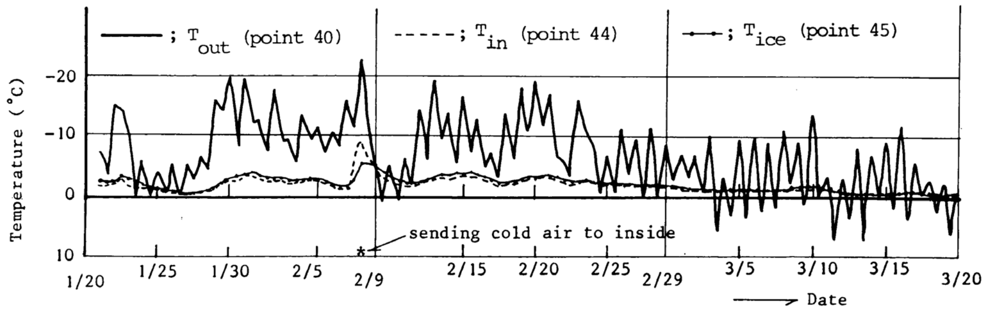

Fig. 1-a Temperature –time curves (Tout:outside air temp., Tin: inside air temp., Tice: ice temp.)

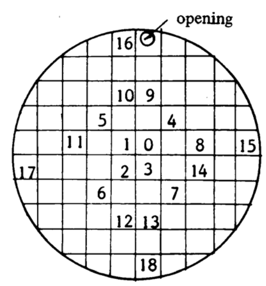

Fig. 1-b Point Nos. for measuring disp.

Measurement of displacement: As shown in Fig.1-b, nineteen locations for measuring displacement were prepared on the inner surface of the dome. The locations 0 to 14 indicate vertical displacement, and 15 to18 indicate normal displacement. The hanging method shown in Fig. 1-c was adopted for easy measurement of vertical displacement of the central section of the dome. Fig.1-d shows displacement-time curves for the central points. On the whole, the displacement in Fig.1-d increases linearly with time from the start (January 20) to the end of February, in what resembles secondary stage, after which it accelerates. The creep deflection of secondary-like stage was evaluated at 2.2mm/day.

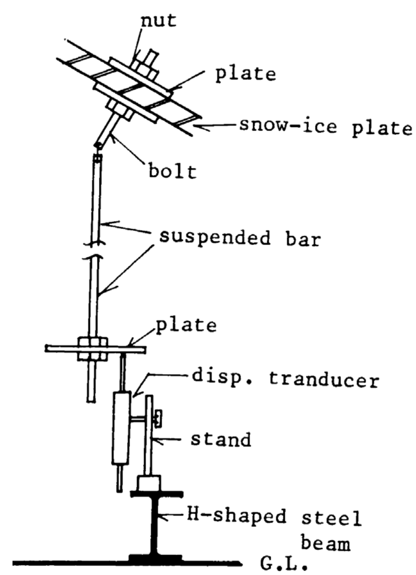

Fig. 1-c Hanging method for measuring vertical displacement

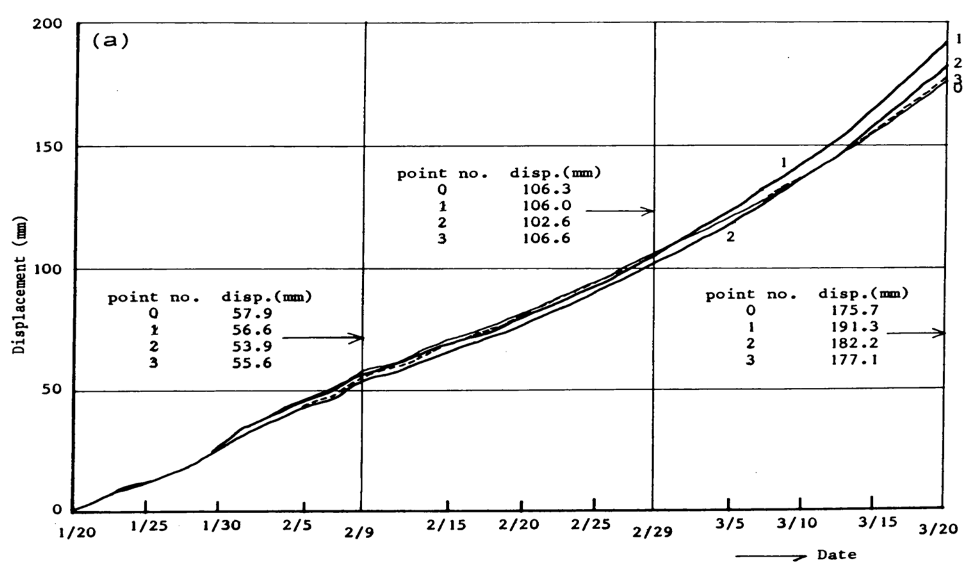

Fig. 1-d Central displacements ($\delta_{0\sim3}$)-time curves

Visual observation up to collapse: After the removal of the automatic measuring equipment on March 20, the structural behavior was observed visually up to the collapse. A large depression was seen near the center. It enlarged rapidly, and at the onset of the collapse, was about 100cm, which is almost 8 times the thickness of the ice.

Snow load: A snow load was applied to the dome, using a snow blower, at the beginning stage. Fig. 1-e shows the distribution of the snow load on February 11. The snow load was considered even and its weight for computation was evaluated at 110kgf/m^2 on average during the displacement measurement period.

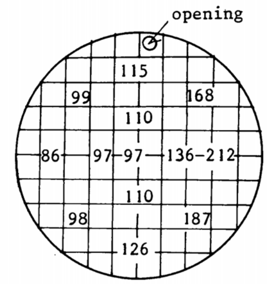

Fig. 1-e Observed snow load (kg/m^2, February 11)

15-m Span Model (Kokawa,1988)

Outline: The construction of a 20-m span ice dome in 1985 (Kokawa and Murakami, 1986) was unsuccessful because too much snow was blown onto the membrane in each application by the snow blower. In terms of creep behavior, the quality of the ice was not satisfactory for structural durability. From this arose the proposal to limit the thickness of the snow to 1cm or less per application. Using this guideline, construction of a 15-m span ice dome was attempted between January 11 and 13 in 1986. In this experiment, each rope was laid orthogonally on the membrane and spaced 1.65m apart in both directions. After completion, a creep test was conducted under a load including that of natural snowfall. Displacement, strain and temperature were recorded automatically every two hours by a programmable data logger from January 21 to March 20, after which the automatic measuring equipment was removed, and the structural behaviour was observed visually until the collapse.

Geometry of dome: The center height and the base diameter of the dome were 313cm and 1370cm, respectively. Assuming the dome to be spherical in shape, its radius was 986cm and its half-open angle was 44^o as calculated from the above data. This dome was shallow compared to today's dome whose height is usually about 450cm.

Measurement of temperature: Three temperature-time curves (Tout, Tin, Tice) were recorded excluding February 28 to March 2 when the data logger was malfunctioning. During this period, Tice was assumed to be almost 0^{o}C. Then, when a daily average temperature of –10^{o}C lasted three days, Tice fell to –5^{o}C. After March 11, the air temperature rose above freezing more frequently, while Tin and Tice gradually rose from –2^{o}C to 0^{o}C leading up to the collapse.

Measurement of displacement and strain: Displacement at 30 locations and strain at 10 locations were measured automatically. The deformation has a generally linear increase with time from the start of recording process to February 28, and the secondary stage creep deflection was evaluated at 1.5 mm/day. After March 4 when the daily average air temperature began to approach 0^{o}C, the deflection began to accelerate. The acceleration continued to increase after the removal of the measuring equipment. The strain at the rope line impressions on February 28 was 0.3% in compression parallel to the impressions, and perpendicular to the impressions, there still seemed to be a small bending moment. According to Mellor and Cole (1983), the maximum strain of polycrystalline ice under a uni-axial creep test is at least 10%. If the ice of the dome has the same capacity as polycrystalline ice, it should be possible to propose a design method considering appropriate allowable strain (or deformation) and evaluating the period how long the ice remains within the allowable strain limit. However, finding the allowable strain is a subject for further study.

Observation by sight until collapse: From March 20 to April 3, Tout fluctuated between plus and minus temperature. During this period, the ice quality deteriorated rapidly as Tice approached 0^{o}C. Just before the collapse, the depression was about 200cm.

Snow load: Snow depth on the dome was measured 5 times at nine places during the measuring period, and the average snow load was evaluated at 20kgf/m^2.

20-m Span Model (Kokawa, 2002a)

Outline: A field experiment on a 20-m span ice dome with a height of 6.5 meters was carried out in Tomamu in both 1999 and 2000. These test domes showed high structural performance compared to the 1985 test dome (Kokawa and Murakami, 1986), which had geometrical and material imperfections because of inexperience in the application of snow. The construction and creep test of the year 2000 dome is described here. It was constructed at the beginning of February. The geometry of the rope cover was determined by Geodesic Triacon division 8 frequencies and the length between nodal points was made a uniform 1.5 meters for ease in production. Displacement was measured at 21 places and temperature was read at 5 places using an automatic device from February 17 to March 27.

Geometry of dome: The finished dome was treated as having the geometry of a spherical surface. The center height of the inner surface and the inner base diameter were 645cm and 1720cm, respectively. The radius of the curvature was computed as 1040cm, and its half open angle was 55.8^o. Fig. 3-b shows the ice thickness at major points. The thickness was inconsistent because all the Styrofoam indicators were covered with ice and not useful in showing the shell thickness during construction. The intended shell thickness was 15cm at the top, 17.5cm at the mid-section and 20cm at the lower-section, but the averaged actual measurements were 12.5cm, 17.9cm and 18.4cm, respectively.

Measurement of temperature and displacement: Temperature-time curves and displacement-time curves for the area near the apex from the completion of the dome to March 27. On March 3, Tout rose above freezing. The next day around noon, it rose rapidly to 5^oC and remained above freezing until the 5th. As a result, both Tin and Tice reached 0^oC, causing melting of the dome's surface. This, in turn, caused a problem in reading the displacement as bolts holding the measuring equipment loosened. It seems that the deformation from March 3 to 5 dramatically increased because of the warm air outside. However, as the deterioration of the ice was superficial, the structural creep performance recovered after March 7, so excluding the period from March 3 to 5, the overall creep deflection was characteristic of the stationary stage during the whole observation period, and it was evaluated at 3.0mm/day.

Observation until collapse: Tout often rose above freezing after March 27. On March 29 and 30, the ice deteriorated rapidly due to the warm air and the strong sunlight. This led to the formation of large depression on the south face of the dome. A large creep deformation occurred just before the collapse. The broken pieces of ice crumbled easily.

Snow load: Snow accumulation during the creep measurement period was recorded in order to evaluate the snow load on the dome. The accumulation was about 20cm during that period with an average snow load of $20kgf/m^2$, which was very small compared to the dead weight of the dome.

30-m Span Model(Kokawa, 2002b)

Outline: A 30-m span ice dome was test-constructed in Tomamu at the beginning of February 2001. The reticular pattern of triangles based on the geodesic Triacon division 8 frequencies of the rope cover. The lengths of the nodal points were determined by spherical trigonometry. The ropes used were 14mm polypropylene. The construction was completed in seven days: two for the foundation work, four for application of snow and water and one for removal of the membrane. Following the construction, a creep test was performed between February 17 and March 23, after which the structural behavior was examined. Five locations for vertical displacement measurement, and 6 locations for temperature measurement: one for outside air, two for inside air and three for ice temperature, were prepaired.

Geometry of dome: The base diameter was 2504cm and the height was 921cm. Therefore, regarding the completed dome as a spherical shell, the radius of the curvature and the half-open angle were 1400cm and 63.4^o respectively. The ice thickness, which was measured at five locations at the upper part of the dome, was in a range of 22cm to 26cm. Readings were taken from ten Styrofoam ice-thickness indicators at the middle and lower parts of the dome. It was found that the thickness at the windward side was from 25 to 30cm, and at the leeward side it was thinner, ranging from 22 to 27cm. Therefore, the average thickness of the completed dome was inferred to be approximately 25cm.

Measurement of temperature: The Tin average was lower and the range wider than that of the smaller domes, because outside air entered through the opening at the windward side. Tice rose gradually toward 0°C from −6°C after March 13.

Measurement of displacement: According to the average displacement-time curve of the 5 locations, whose displacement readings were very similar to each other during the period of recording, a roughly 150mm downward displacement from March 20 to 21, a roughly 190mm upward displacement from March 21 to 22, and other unexpectedly large changes of displacement occurred throughout the recording period. However, it is doubtful that these readings reflect the true displacement behavior. Possible influences, such as the thermal expansion of the ice or the chain used in the measuring apparatus, temperature-induced fluctuations in the snow floor level and the crack near the opening were investigated; but these factors did not seem to give any rational reason for the phenomenon. In addition, the displacement transducers were tested for the influence of temperature, but a clear connection to the phenomenon was not found here either. The average daily creep displacement as a straight line was derived from the actual data to which the least square method is applied. The slope of this straight line representing the average creep displacement was computed at 6.5mm/day.

Behavior of collapse: After removing the displacement transducers on March 23, the structural behavior was visually observed until the dome's collapse. After April 2, the outside air temperature often exceeded 0˚C. According to an on-site observer, the weather conditions promoted rapid melting of the ice. This was accompanied by a rushing sound that echoed inside the dome. Then, following three consecutive days with air temperature averaging over 0°C, the collapse occurred on April 10, at about 1 p.m. The quality of the broken pieces of ice at the south side was very poor due to solar damage. Although the behavior was "brittle," since a large deformation did not appear before the collapse, the dome maintained considerable structural stability, even as temperatures rose. Clearly, it was the spring weather condition that lowered the quality of ice plate and led to the brittle failure of the structure in this case.

Snow load: As the snow accumulation was not actually measured at the construction site, meteorological data from a near by government recording site was used for the evaluation of snow load on the test dome. According to this data, the increase in snow accumulation from February 16 to March 24, 2001 was 17cm and was evaluated at 34kgf/m^2 of snow load with an assumed density of 0.2gf/cm^3. However, the half load, 17kgf/m^2, which is the average load for the time period, is the value to use in calculating the viscosity of ice as described later in this paper.

Creep Deflection-Time Curve of Ice Dome

According to the field experiments described in the previous sections, the ice temperature of the domes is between 0 °C and –5 °C. The ice in this range creeps easily, and the deflection of the structure increases rapidly with time, even if there is no increase in working stress. Therefore, it is necessary to prepare a rational structural design method considering not only a stress regulation but also deformation or strain rules in relation to time. From the results of the past field experiments of ice domes spanning 10 to 30-meters, the following are pointed out in relation to the deflection-time curves, including the structural behavior up to the failure:

1. the creep deflection has a linear function of time at the beginning
2. the deflection rate increases with time until collapse
3. collapse occurs after the daily average temperature above freezing lasts 2 to 3 days
4. when the deterioration of the ice is not too advanced, a large deformation should be visible before the collapse.

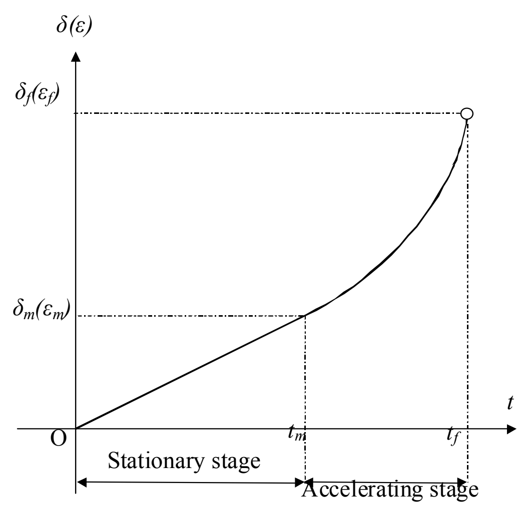

Fig.2. Model of creep deflection-time curves

The simplified deflection-time curve is shown in Fig. 2. That is, the creep deflection-time curve is approximately given by connecting two lines; the first is secondary stage where the deflection has a linear function of time, and the second is accelerating stage where the deflection rate increases with time up to the point of failure. This curve coincides with a strain-time curve from which the elastic response and the primary stage in the uniaxial creep test under constant stress and temperature are omitted. While the quantitative evaluation of $\delta_m(\varepsilon_m)$ and $\delta_f(\varepsilon_f)$, which is indispensable in establishing an allowable-strain design method, is left for future study, the following section proposes a formula for creep deflection in an ice dome in secondary stage.

SIMPLIFIED FORMULA FOR STATIONARY CREEP DEFLECTION OF ICE DOME

A simplified formula for stationary creep deflection of an ice dome is proposed in this section. The geometry of the dome is complicated because the pneumatic formwork consists of a double-planed membrane bag and a rope cover (Kokawa, 1985), and it is technically difficult to achieve the intended ice thickness by the method of blowing snow and spraying water. For these reasons, the numerical modeling for the structural analysis is extremely difficult, and the application of the finite element method is not practical. Therefore, the geometry of the dome is treated simply as a partial sphere with even shell thickness so that the simple solution in the membrane shell theory can be used. The derivation of the formula is as follows: assuming the ice to be a Newtonian fluid, and using the invariant theory of creep material and the membrane theory of a thin shell, the vertical deflection at the top is derived analytically. Evaluation of the viscosity is based on the creep deflection data from past field experiments.

Creep Model of Ice

As described in introduction in this paper, the maximum compressive membrane force in the structure is less than 1 kgf/cm^2 . The compressive strength of the ice produced by blowing snow and spraying water is expected to be at least 40 kg/cm^2 or more under short-term loading, so the designed dome should be strong enough to support the load of its own weight. However, as found in past field experiments, the ice temperature usually remains in the range of 0^o to −5^{o}C, and in this range, the ice creeps easily with time even though the working stress is small. The subsequent large deformation renders the dome unusable as an architectural structure and causes instability leading to collapse. Therefore, it is necessary to prepare a rational structural design method that takes into consideration not only stress regulation but also deformation or strain rules in relation to time. There are many creep studies on snow and ice such as fresh ice (Jacka, 1984;Mellor and Cole, 1983), sea ice(Rene Tinawi and Jean-Robert Murat,1978), spray ice(Shields et al., 1989)and snow(Shapiro *et al.*, 1997). There is, however, little information on the creep data for the ice used in ice shell construction under the uniaxial test, although this ice may belong to T1 snow ice which is considered to be isotropic (Michel, 1978). In this study, the creep model for the ice is assumed to be a Newtonian fluid as given in eq.(1).

$$\sigma = \eta\dot{\varepsilon} \qquad (1)$$

where σ is uniaxial stress, $\dot{\varepsilon}$ is uniaxial strain rate and η is viscosity. Although η should be treated as the function of ice temperature in Arrhenius equation, η is evaluated as constant, in the range of 0^o to −5^{o}C. This value of η is based on the data from past field experiments.

Solution of Creep Deflection

Assuming that a spherical dome (Fig.3) is subjected to a dead type of load whose magnitude per unit area is constant and equal to q, the normal membrane stresses are

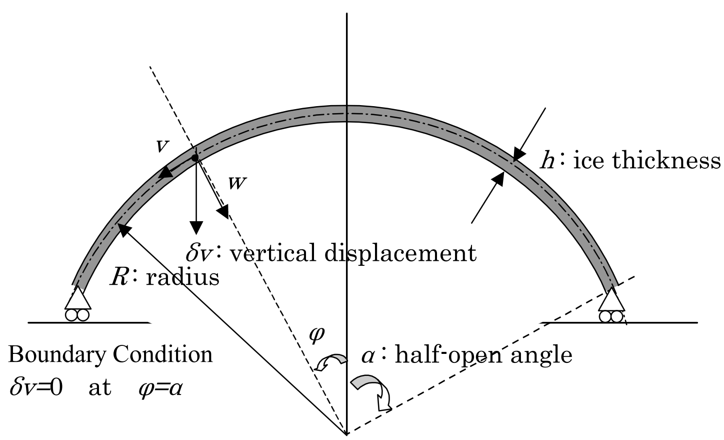

Fig. 3 Parameters of ice dome for analysis

given by Eq.(2)(Timoshenko and Woinoesky-Krieger, 1959):

$$\sigma_{\varphi} = -\frac{qr}{h(1+\cos\varphi)}, \qquad \sigma_{\theta} = -\frac{qr}{h}\left(\cos\varphi - \frac{1}{1+\cos\varphi}\right) \tag{2}$$

where σ_{φ} is membrane stress in the meridian direction, σ_{θ} is membrane stress in the parallel direction, *h* is even shell thickness, *r* is the radius of the shell and φ is the angle made by a normal to the dome's surface and the axis of rotation. If *q* is the dome's own weight only, these stresses are expressed without *h*. Using the invariant theory of ice as a creep material, the vertical deflection is derived analytically. The evaluation of the viscosity in the ice is based on the creep deflection data from past field experiments. Strain-displacement relation is shown in Eq.(3).

$$\varepsilon_{\varphi} = \frac{1}{r}\left(\frac{dv}{d\varphi} - w\right), \qquad \varepsilon_{\theta} = \frac{1}{r}(v\cot\varphi - w) \tag{3}$$

where *w* is displacement in the normal direction, *v* is displacement of the meridian direction, ε_{θ} is strain in the parallel direction and ε_{φ} is strain in the meridian direction. The relationship between the strain and the membrane stress is drawn by using the invariant theory (Odqvist and Hult, 1967). This is shown in Eq.(4).

$$\eta\dot{\varepsilon}_{\varphi} = \sigma_{\varphi} - 0.5\sigma_{\theta}, \qquad \eta\dot{\varepsilon}_{\theta} = \sigma_{\theta} - 0.5\sigma_{\varphi} \tag{4}$$

Eliminating *w* in Eq.(3), and using Eq.(2) and (4), Eq.(5) forms the first order of ordinary differential equation for $\dot{v}$.

$$\frac{d\dot{v}}{d\varphi} - \dot{v}\cot\varphi = \frac{1.5qr^2}{\eta h}\left(\cos\varphi - \frac{2}{1+\cos\varphi}\right) \tag{5}$$

The solution is expressed in Eq.(6)

$$\dot{v} = \frac{1.5qr^2}{\eta h}\sin\varphi\left(\log(1+\cos\varphi) - \frac{1}{1+\cos\varphi}\right) + C\sin\varphi \tag{6}$$

where *C* is an arbitrary constant, and given by using a boundary condition that the vertical displacement δ_v becomes 0 where $\varphi = \alpha$. δ_v is then expressed by Eq.(7).

$$\dot{\delta}_v = \dot{w}\cos\varphi + \dot{v}\sin\varphi = \left(\dot{v}\cot\varphi - r\dot{\varepsilon}_{\theta}\right)\cos\varphi + \dot{v}\sin\varphi = \frac{1}{\sin\varphi}\dot{v} - \frac{r}{\eta}\cos\varphi\left(\sigma_{\theta} - 0.5\sigma_{\varphi}\right) \tag{7}$$

Substituting Eq.(6) and (2) into Eq.(7),

$$\dot{\delta}_v = \frac{qr^2}{\eta h}\left\{1.5\log(1+\cos\varphi) + \cos^2\varphi - 1.5\right\} + C$$

Finally, the creep deflection is given by Eq.(8).

$$\dot{\delta}_v = \frac{qr^2}{\eta h}\left\{\left(\cos^2\varphi - \cos^2\alpha\right) + 1.5\log\left(\frac{1+\cos\varphi}{1+\cos\alpha}\right)\right\} \tag{8}$$

Placing φ=0 into Eq.(8), the deflection at the apex becomes Eq.(9).

$$\dot{\delta}_{v0} = \frac{qr^2}{\eta h}\left\{\left(1 - \cos^2\alpha\right) + 1.5\log\left(\frac{2}{1+\cos\alpha}\right)\right\} \tag{9}$$

Evaluation of Viscosity

Viscosity is calculated by placing the experimental results of each model into Eq.(9). This is shown in Table 1. As seen here, the viscosity of the 15-m model is evaluated at 4534 kgf·cm^{-2}·day, which is a very large value compared to the others. The reason for this is not yet clear, so as a precaution, this large value is omitted from the averaging for use in the simplified formula for creep deflection. The exact average of the other three models is 3527 kgf·cm^{-2}·day, but 3500 kgf·cm^{-2}·day is substituted for η as the appropriate viscosity value in Eq.(9). The following equation is given as the simplified formula for creep deflection at the apex of an ice dome.

$$\dot{\delta}_{v0} = \frac{qr^2}{3500h}\left\{\left(1 - \cos^2\alpha\right) + 1.5\log\left(\frac{2}{1+\cos\alpha}\right)\right\} \tag{10}$$

Referring to Eq.(10), it is easy to calculate $\dot{\delta}_{vo}$ because it is a function of r (cm), h (cm), α(°) and q(kgf/cm^2). For example, if the dome has a 40m base diameter and a height of 12.35m, $\dot{\delta}_{vo}$ is calculated at 15.5 mm/day(=1.55 cm/day) under dead load, substituting r=22.37m (=2237 cm), α=63.4°, $q/h=\rho$ =0.85 g/cm^3 (=0.85·10^{-3}kg/cm^3) into Eq.(10).

Table 1 Calculated viscosity

Model	Radius	Thickness	Half-open angle	D. Load	Snow L.	Total L.	σ_{top}	Creep deflection	Viscosity
(m)	R(cm)	h(cm)	α (°)	(kg/m^2)	(kg/m^2)	w(kg/m^2)	σ (kg/cm^2)	δv (cm/day)	η(kgcm^{-2}day)
10	765	12.5	44.8	106	110	216	0.66	0.22	3364
15	986	14.5	44	123	20	143	0.49	0.15	4534
20	1040	17.9	55.8	152	20	172	0.50	0.30	3654
30	1400	25	63.4	213	17	230	0.64	0.65	3563

CONCLUDING REMARKS

Based on past field experiment data from ice domes spanning 10 to 30-m, the creep deflection-time curve is approximated by connecting two lines; the first line represents the stationary stage in which the deflection is a linear function of time and the second represents the accelerating stage in which the deflection rate increases with time up to failure. This paper proposes a simplified formula for computing the creep deflection during the stationary stage. The derivation of the formula is as follows: assuming ice to be a Newtonian fluid, and using the invariant theory for the creep material and the membrane theory for a thin shell, the vertical deflection at the apex is analytically derived. The evaluation of the ice viscosity is based on creep deflection data from past field experiments.

It is the goal of this study to establish a rational structural design method that takes into consideration not only stress regulation but also deformation or strain rules in relation to time. The simplified formula proposed in this paper is one step toward that goal.

REFERENCES

Kokawa, T. and Hirasawa, I., 1982/1983. On the Behavior of Ice Domes under Short-term Loading (in Japanese), *Bull. Hokkaido Tokai Univ.,* Vol.3/4, 77-85.

Kokawa, T., 1985. Experimental Studies on Ice Shell in Asahikawa. Cold Regions Science and Technology, 11, 150-170.

Kokawa, T. and Murakami, K., 1986. Challenge to 20-m Span Ice Dome, Shells, Membrane and Space Frames, Proceeding IASS Symposium Osaka, Vol.1, 297-304.

Kokawa, T., 1988. Construction and Creep Test of 15-m Span Ice Dome. Proceeding of IAHR Ice Symposium in Sapporo, 90-399.

Kokawa, T., Itoh, O. and Watanabe, T., 2002a: Re-Challenge to 20-m span ice dome, CD-ROM of IASS in Nagoya, TP187.

Kokawa, T., 2002b: Field Study of a 30-m Span Ice Dome, Journal of IASS, Vol. 43 (2002) n.2, pp.93-100.

Kokawa, T., 2005. State of the Art Development in Ice Shell Construction. Proceeding of 17th Canadian Hydrotechnical Conference (CD-ROM), Edmonton, 973-982.

Malcolm Mellor and David M.Cole, 1983. Stress/Strain/Time Relations for Ice under Uniaxial Compression. Cold Regions Science and Technology, 6, 207-230.

Bernard Michel, 1978. Ice Mechanics. 72-73. Les Presses De L'Universite Laval, Quebec.

T.H, Jacka, 1984. The Time and Strain Required for Development of Minimum Strain Rates in Ice. Cold Regions Science and Technology, 8, 261-268.

D.H.Shields and L.Domashuk, E. Funegard, B.H.Kjartanson and F.Azizi, 1989. Comparing the Creep Behaviour of Spray Ice and Polycrystalline Freshwater Ice. Eighth International Conference on Offshore Mechanics and Arctic Engineering, The Hargue-March 19-23, 235-245.

Rene Tinawi and Jean-Robert Murat, 1978. Sea Ice-Flexural Creep. Proceedings of International Association of Hydraulic Research (IAHR) International Symposium on Ice Problem, Lulea, Sweden, August 7-9, Part 1, 49-75.

Lewis H.Shapiro, Jerome B. Johnson, Matthew Sturm, and George L. Blaisdell, 1997. Snow Mechanics, Review of the State of Knowledge and Applications. CRREL REPORT 97-3, 35pp.

Timoshenko,S. P. and Woinoesky-Krieger,S., 1959. Theory of Plates and Shells. McGraw-Hill, London, 2nd Ed., 436-447.

Odqvist and Hult (translated by Sumio Murakami into Japanese, 1967). Theory of creep mechanics, Baihukan, 309pp.