STRUCTURAL IDEA OF RETRACTABLE LOOP-DOME

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SUMMARY

The author proposes a 3-dimensional multi-angulated scissor element as the main structural element of a new type of dome called 'Retractable Loop-Dome'. It allows the dome to continuously change its shape according to changes in the diameter of the oculus. This paper describes 1) the geometry of the 3-D multi-angulated scissor element determined by cutting a sphere with a plane passing through the apex, 2) a proposal for a rational structural system with expandable circular rings for a large span and 3) a numerical simulation of the changing shape and the trial construction of an aluminum model with a 10 m base diameter.

Keywords: *Retractable Loop-Dome, 3-Dimensional Multi-Angulated Scissor Element, Rational Structural System, Expandable Circular Ring.*

1. INTRODUCTION

The retractable loop-dome has several architectural advantages such as:

1) the structural efficiency of a shell,
2) environmental control by changing the diameter of the oculus,
3) rational construction utilizing peripheral folding, and
4) artistically dynamic expression through change in overall shape.

However, such a dome has not yet been realized because there still are some problems relating to the structural rationality.

There are two well-known ideas using similar structural elements for retractable loop-dome. The "Angulated Element" used in C. Hoberman's "Iris Dome", has an ingenious geometrical property [1], [2], [3]. This pioneering idea is greatly valuable in providing visual images. Z. You and S. Pellegrino proposed "Generalized Multi-Angulated Element" which is the member of a two-dimensional foldable structure. Then they proposed the interesting idea of projecting the elements onto a curved surface to simulate a retractable loop-dome of a required shape [4], [5].

These ideas, though the kinetic conditions are theoretically satisfied, have obstacles to overcome, relating to the rationality of structural details, such as the joints. C. Hoberman's angulated element must be pin-connected individually in a plane; therefore it may be assumed that the joint has disadvantages such as difficult production and reduction in joint stability. Z. You and S. Pellegrino's connectors between multi-angulated elements must be perpendicular to the plane of projection because of the theoretical requirement; therefore problems may arise relating to both the production of connectors and the structural strength, particularly in the case of a deeply curved surface.

While studying the two ideas, the author discovered a 3-dimensional multi-angulated scissor element which solves the above-mentioned problems [6]. This paper describes:

1) the geometry of the 3-D multi-angulated scissor element determined by cutting a sphere with a plane passing through the apex,
2) a proposal of a rational structural system for a large span, and
3) a numerical simulation of the changing shape and the trial construction of an aluminum model with a 10 m base diameter.

2. 3-DIMENSIONAL MULTI-ANGULATED SCISSOR ELEMENT

2.1. Determination of Geometry

The geometric form of the 3-dimensional multi-angulated scissor element is determined by cutting a sphere with a plane. The scissors' hinge-points (1, 2, ..., i, i+1, ..., n) of the element are arranged on the surface of a sphere S as follows:

a) Cutting sphere S with inclined plane P that intersects apex T, as shown in Fig. 1(a).

b) Arranging hinge-point on circle Q (ellipse on the XY plane), as shown in Fig. 1(b). That is,

$\theta_{12} = \theta_{23} = \ldots = \theta_{i(i+1)} = \ldots = \theta_{(n-1)n}$ on XY plane.

The following description explains the reason why this element can move rigidly without elastic deformation on an axisymmetrically curved surface.

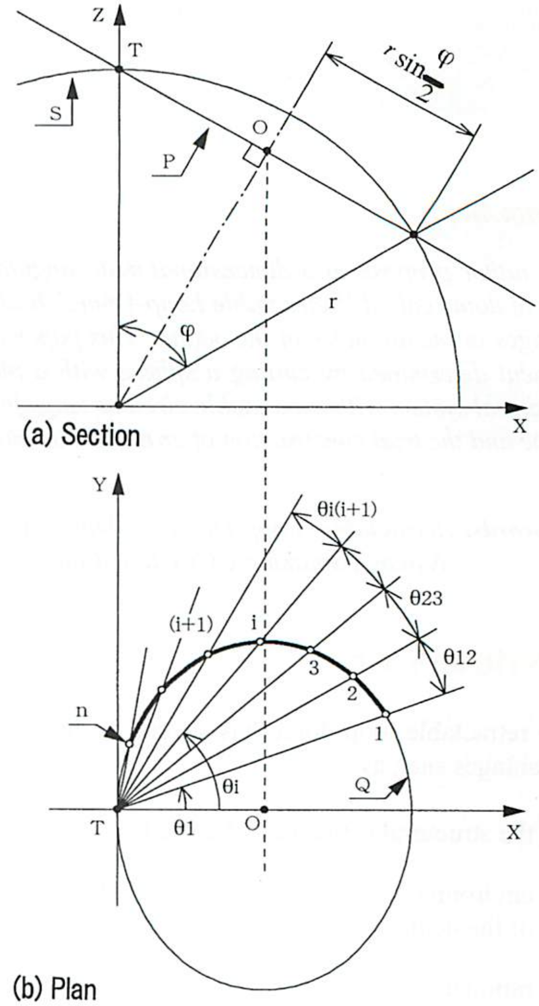


Figure 1. Arrangement of hinge-points

Proposition:

As shown in Fig. 2, there are two different points T and A on a circle O. The end point O of the line OA moves on the circle T, and the other end point A moves on the straight line TA. O' and A' are defined as moving points, respectively. In this situation, the intersecting angle between the line OA and O'A' is equivalent to ∠OTO' and independent from the location of point A.

Demonstration:

In order to demonstrate the proposition, it shows here that under the condition ∠OTO'=t, ∠OTA=a, OA=O'A'=OT=O'T, ∠A'MA is equal to ∠OTO'(=t).

∠OAT=∠OTA=a (because ΔOAT is an isosceles triangle).

∠O'A'A=∠O'TA=a-t (because ΔO'A'T is an isosceles triangle).

Therefore, ∠A'MA=∠OAT-∠O'A'A=a- (a-t)=t

Finally, it shows exactly the above-mentioned proposition.

The following theorem is derived from this result.

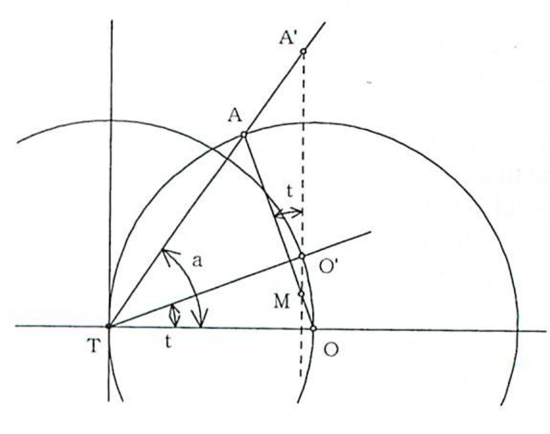


Figure 2. Proposition and demonstration

Theorem:

There are four different points T, A, B and C on the circle O, as shown in Fig. 3. The end points A and B on the arc AB move to the points A' and B' on the lines TA and TB, respectively, then an arbitrary point C on the arc AB moves to the point C' on the line TC, and the center point O' of the arc A'B' is located on the circle T. That is, when translating the arc AB from the point O to the point O' and rotating ∠OTO' around the point O', then the point C moves on the straight line TC.

As shown in Fig. 1, the hinge-points (1, 2,..., i,..., n) of the 3-dimensional multi-angulated scissor elements are arranged on the circle Q. As a result of the above-mentioned theorem, it indicates that the element can move rigidly on a 3-dimensional axisymmetric surface without elastic deformation, and a family of these elements can basically produce a retractable loop-dome by their lamella arrangement.

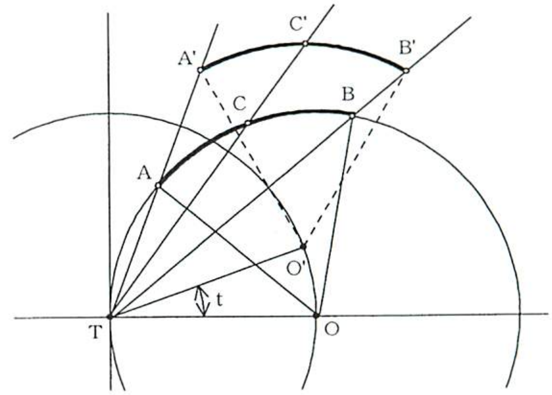


Figure 3. Theorem

2.2. Intersecting Angle between Hole-Axes
(see Appendix)

The scissors' hinge-axis of all elements coincides with the normal direction of one sphere S at the reference state shown in Fig. 1. However, during the retraction, departing from the reference state, the hole-axis of each element goes toward the center of each sphere. Therefore, a small variation in angle is caused by a slight difference in direction between the scissors' hinge-axis and the hole-axis. A loose-hole or an embedded spherical roller bearing (or self-aligning ball bearings) at the hinge-point shown in Fig. 4 may be needed in order to absorb the difference.

2.3. Feature

As a result of 2.1 and 2.2, an element based on this idea is expected to have the following advantages:

1) Smooth transmission of member forces.

2) Easy manufacture of element and hinge-detail.

3. PROPOSAL OF STRUCTURAL MODEL

3.1. Expandable Circular Ring

As described in 2, the 3-dimensional multi-angulated scissor element is used as the main structural element for a retractable loop-dome, which can change the diameter of the oculus, by its lamella arrangement on a sphere. In order to apply such a dome in practice, the unstable structure must be changed to a stable structure. The problem to be discussed here is how to establish a rational structural system for a large span, considering the retraction technology of today. Then, as shown in Fig. 5, a system is proposed, in which an expandable ring is added to both the inner and outer circles of the dome, so as to produce the structural efficiency of shell-like behaviour. Each expandable circular ring consists of expandable rods that form a regular polygon. In the case of the outer ring acted in tension, such a rod may in practice be possible to make by using electrical actuation technology because of its small expansion-traction ratio. On the other hand, in the case of the inner ring, an expandable rod may be difficult to make because of large expansion-traction ratio and action in compression. Fig. 6 sketches an example of such a rod.

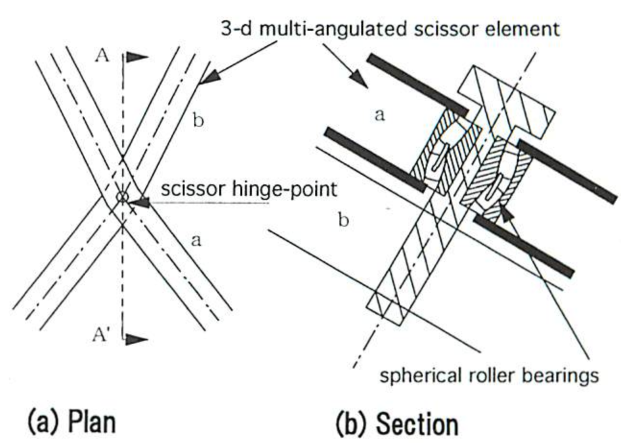


Figure 4. Sketch of hinge-point

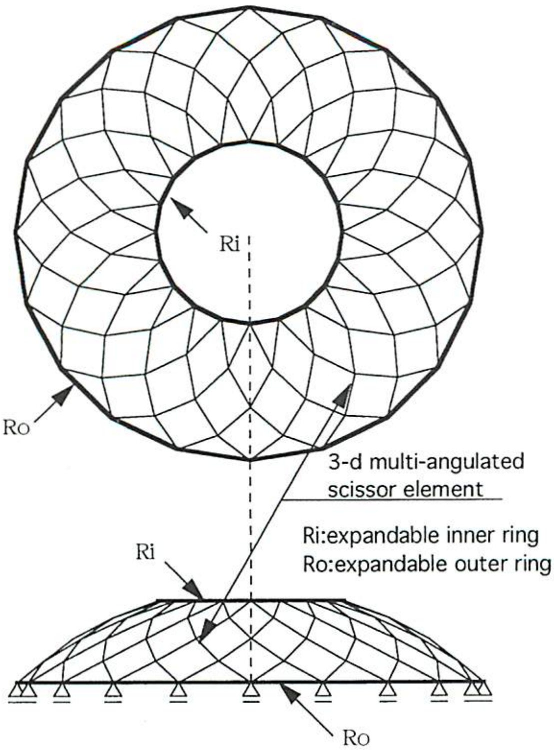


Figure 5. *Expandable ring for structural rationality*

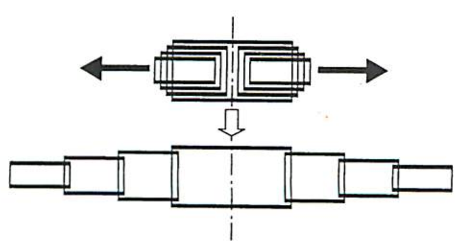


Figure 6. *Multi-overlapped cylinder*

3.2. Simulation of Changing Shape
(see Appendix)

It is possible to produce a structural model of the new type of retractable loop-dome by combining the structure (upper structure) described in 3.1 and a one-layer truss structure (lower structure) inserted between the outer ring and the foundation ring. In this model, the truss structure is pin-connected to both rings, and supposed to have a mechanism of changing geometry. An example of a simulation of this model is shown in Fig. 7. Here, the geometric parameters of the upper structure are φ=55°, θ_1=38°, n=5 and θ_5=74° referring to Fig. 1, D_1 (the diameter of the outer ring) equals to D_0 (the

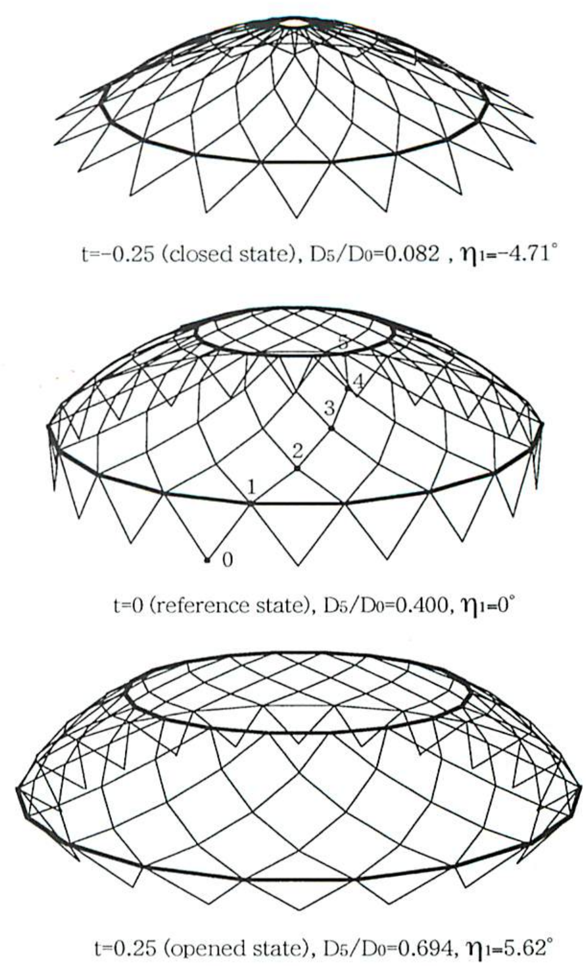
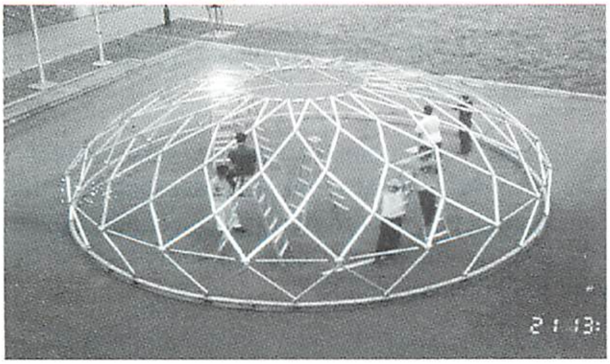


Figure 7. *Simulation of changing shape*

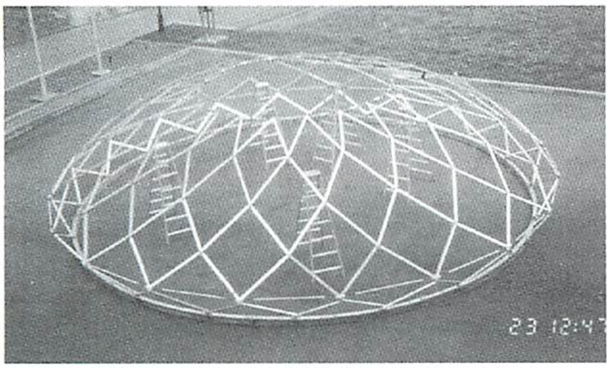
diameter of the foundation ring) at the reference state, and ξ (the parameter of the truss; see Appendix) is 55°. As seen in Fig. 7, the diameter of this oculus boldly changes in size, and the entire structure makes a dramatic change in its geometric shape. This dome may give dynamic expression never before seen in usual domes.

3.3. Trial Construction

In order to investigate both the structural and constructional problems of this system, an aluminum model with a 10 m base diameter was constructed. Fig. 8 shows the condition of the dome when the oculus is at its minimum and maximum diameter. Aluminum rectangular pipe (□ 20×40×2) was welded to form the 3-dimensional multi-angulated scissor member. Special manual rods



a) Minimum oculus diameter



b) Maximum oculus diameter

***Figure 8**. Aluminum model with 10 m base diameter*

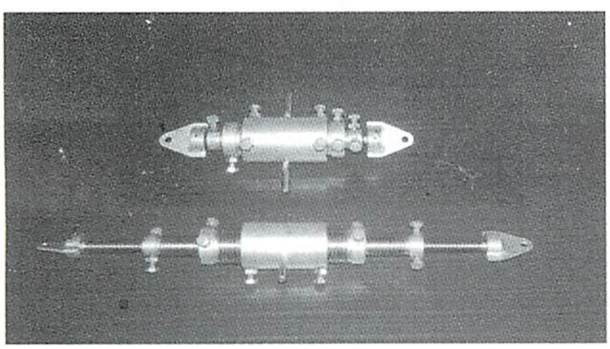


***Figure 9**. Expandable inner compression rod*

were used as the expandable rods. The tension-rod on the outer side consisted of steel pipe (ϕ 13.8×2.3) and a long screw bolt M8. The compression-rod was manufactured mechanically, based on the idea shown in Fig. 6, and it changed from 34 cm to 67 cm in length (Fig. 9). Holding a certain relationship in length of these rods could change the whole shape of this model. The experiment showed that the dome could not support its own weight without these rings, thus confirming that the rings are indispensable members of this rational structural system.

4. CONCLUSION

First, this paper described the geometrical reason why this retractable loop-dome can be composed of 3-dimensional multi-angulated scissor elements determined by cutting a sphere with planes passing through the apex. Secondly, it proposed a rational structural system with expandable circular rings for a large span. Thirdly, it visually showed the simulation concerning the changing geometry of a structural model with a one-layer truss structure, and explained the trial construction of an aluminum dome model with a 10 m base diameter.

The loading test of this model and an investigation by numerical and experimental analysis of the structural behaviour are planned.

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APPENDIX

Referring to Fig. 1, mathematical expressions for the graphic simulation shown in 3.2 are as follows.

a) 3-dimensional coordinate of scissors' hinge-point

x, y, z coordinates at the point i are given in Eq. (1)

$$x_i = \frac{\sin\varphi\left\{\cos t + \cos\left(\frac{\varphi}{2}\right)\cdot\sin t\cdot\tan\theta_i\right\}r}{1+\cos^2\left(\frac{\varphi}{2}\right)\cdot\tan^2\theta_i}$$

$$y_i = \tan\theta_i\, x_i \qquad (1)$$

$$z_i = r - \tan\left(\frac{\varphi}{2}\right)x_i$$

Here, t is a parameter for changing shape, and corresponds to the t in Fig. 3. Eq. (1) is derived from the intersecting point between the ellipse with the central point O' and the straight line passing through the apex.

b) Angle between the hinge-axis and the hole-axis

Each hinge point is located on not only the circle Q, but also on the sphere S with its radius r. x, y, z coordinates at the center point P' of the sphere S are given as Eq. (2).

$$p_x = \frac{r\cdot\sin\varphi}{2}(\cos t - 1)$$

$$p_y = r\cdot\sin\left(\frac{\varphi}{2}\right)\cdot\sin t \qquad (2)$$

$$p_z = r\cdot\sin^2\left(\frac{\varphi}{2}\right)(1-\cos t)$$

Substituting t = 0 at the reference state into Eq. (2), $p_x = p_y = p_z = 0$

The sphere point-point of each scissor element is different from others except p_z, under the retraction. Considering this feature, the intersecting angle η_i between the hinge-axis and the hole-axis at the point i is given in Eq. (3).

$$h_i = \frac{\left\{\sin\varphi(1-\cos t)\tan\theta_i + 2\sin\left(\frac{\varphi}{2}\right)\cdot\sin t\right\}}{2\sqrt{1+\tan^2\theta_i}} \qquad (3)$$

$$\eta_i = \sin^{-1} h_i$$

c) Geometric parameter ξ of truss structure

The geometric parameter ξ is shown in Fig. A. It is easy to numerically pursue the changing geometry of the dome by considering the compatibility on the boundary between the upper and lower side of the truss structure.

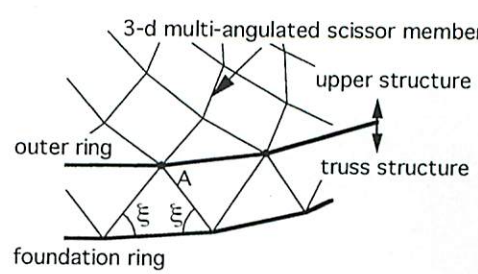

Figure A. Geometric parameter